Indian Journal of Production and Thermal Engineering (IJPTE)
ISSN: 2582-8029 (Online), Volume-6 Issue-4, June 2026

Experimental Study and Numerical Simulation of Green Tea Drying Kinetics using a Heat Pump System Integrated with Auxiliary Electric Heating

Ngo Quang Truong, Nguyen Thi Viet Linh, Pham The Vu, Pham Van Duy

Abstract: *Controlling the moisture removal process in Thai Nguyen green tea drying is a significant challenge due to the complex capillary-porous structure of the leaves and the high thermal sensitivity of bioactive compounds. This study proposes an advanced hybrid drying solution that combines heat pump technology (HPD) with auxiliary electric heating to optimise drying kinetics and energy efficiency. In addition to the experimental investigation of three multi-stage temperature-control modes, a mathematical model based on 1D moisture-diffusion partial differential equations (PDEs) was developed. This model was solved using the implicit finite difference method (FDM) in MATLAB to accurately describe the moisture concentration gradient from the core to the material surface in real time. The experimental and simulation results show high compatibility (RMSE < 0.05). The data indicate that the stepped drying mode (35–40–45°C) is the optimal strategy, reducing the drying time by 27.2% compared to the conventional constant-temperature heat pump method. This strategy ensures uniform moisture distribution and effectively prevents surface "case hardening." Among the six thin-layer kinetic models evaluated, the Midilli & Kucuk model exhibited superior performance, with a coefficient of determination (R2) of 0.9981 and an RMSE of 0.00179. The novelty of this research lies in establishing a "thermal safety zone" through a multi-stage heating mechanism, which maintains a maximum drying rate of 0.035 g/min during the critical falling-rate period. The findings confirm that PDE-based numerical simulation is a fundamental tool for optimising hybrid drying systems, enabling accurate prediction of the drying endpoint and establishing temperature profiles to preserve sensitive bioactive substances in industrial applications.*

Keywords: Green tea Drying, Hybrid Heat Pump, Auxiliary Heating, Numerical Simulation, Drying Kinetics, Multi-Stage Drying.

Manuscript received on 11 April 2026 | First Revised Manuscript received on 16 April 2026 | Second Revised Manuscript received on 17 May 2026 | Manuscript Accepted on 15 June 2026 | Manuscript published on 30 June 2026.

*Correspondence Author(s)

Ngo Quang Truong, School of Electrical and Electronic Engineering, Hanoi University of Industry, 298 Cau Dien, Tay Tuu, Hanoi, Vietnam. Email ID: ngotruong2366@gmail.com

Nguyen Thi Viet Linh*, Faculty of Energy Engineering, School of Electrical and Electronic Engineering, Hanoi University of Industry, 298 Cau Dien, Tay Tuu, Hanoi, Vietnam. Email ID: linhntv@haui.edu.vn, ORCID ID: 0009-0009-0287-1022

Pham The Vu, Faculty of Energy Engineering, School of Electrical and Electronic Engineering, Hanoi University of Industry, 298 Cau Dien, Tay Tuu, Hanoi, Vietnam. Email ID: vupt@haui.edu.vn

Pham Van Duy, Institute of Energy Science and Environment (IESE), A9 Building, 18 Hoang Quoc Viet, Cau Giay, Hanoi, Vietnam. Email ID: phamvanduy.nd18@gmail.com

Nomenclature:
FDM: Finite Difference Method
HPD: Heat Pump Drying
PDEs: Partial Differential Equations
MR: Moisture Ratio
DR: Drying Rate
RMSE: Root Mean Square Error

I. INTRODUCTION

Green tea (*Camellia sinensis*) has long been recognized as a beverage of high economic and medicinal value due to its rich content of health-promoting bioactive compounds [1]. In Vietnam, Thai Nguyen province—with its characteristic low-mountain terrain, fertile soil, and cool climate—has become a key tea-growing region, providing products with distinctive flavours for both domestic and international markets [1]. The chemical composition of green tea leaves is diverse, comprising polyphenols (30–35%), proteins (15–20%), amino acids (1–4%), and other essential minerals [2].

However, the organic compounds in tea are highly sensitive to environmental temperature and humidity. During processing, imprecise control of heat and moisture parameters often leads to colour degradation and a severe decline in both sensory quality and bioactive content. Consequently, applying modern drying technologies to optimise moisture removal kinetics and preserve product quality is urgent.

[Fig.1: Green Tea Leaves Used in the Study]

Heat Pump Drying (HPD) is an advanced technology that enables drying at low temperatures and low relative humidity of the drying agent, helping to maintain better colour and flavour than traditional thermal drying. drying methods [3], [4]. Nevertheless, pure HPD is often limited by slow drying rates during the falling-rate period due to low internal diffusion driving forces. To overcome this drawback, integrating HPD with Auxiliary Electric Heating has emerged as a promising research trend [5]. This hybrid solution leverages the heat pump's temperature stability in the initial stage and the electric heater's flexible temperature control in the later stage to enhance moisture diffusion from the material core.

Despite experimental studies demonstrating the effectiveness of hybrid drying systems, understanding the heat- and mass-transfer mechanisms within the material remains a significant challenge. The spatial and temporal variations in moisture concentration within the tea leaf structure are difficult to observe directly in experiments. Therefore, the application of Numerical Simulation to solve systems of Partial Differential Equations (PDEs) that describe diffusion processes is a key tool for accurately predicting drying kinetics.

This study aims to evaluate the effects of different hybrid drying modes on the drying kinetics of Thai Nguyen green tea leaves. The highlight of this research is the development of a mathematical model based on the Finite Difference Method (FDM) to simulate the moisture concentration gradient within the material. Experimental data were validated against the numerical model and six common thin-layer mathematical models (Page, Wang and Singh, Newton, Midilli and Kucuk, Two-term, and Verma) to establish the optimal moisture transformation law. The research results not only provide a scientific basis for optimising processing procedures but also contribute to enhancing the economic value of local agricultural products through precise control of drying technology.

II. RESEARCH METHODOLOGY AND MATHEMATICAL MODELING

A. Determination of Moisture Parameters

The initial moisture content of green tea leaves (M_o) was determined by drying the samples at 105 °C in an electric oven for 6 hours to constant weight. The moisture content at time t (M_t) during the experimental process was calculated based on the weight change of the drying material using Equation (1) [6]:

$$M_t = \frac{W_t - W_d}{W_t}.100\% \qquad (1)$$

Where: W_t is the mass of the material at time t (g); W_d is the absolute dry mass of the material (g), and M_t is the moisture content on a dry basis (kg/kg db).

To standardise the experimental data for mathematical modelling, the dimensionless moisture ratio (MR) was used. Given the extended drying time and the fact that the equilibrium moisture content (M_e) of green tea is significantly smaller than M_o and M_t, the MR calculation formula was simplified as follows in Equation (2):

$$MR = \frac{M_t - M_e}{M_o - M_e} \approx \frac{M_t}{M_o} \qquad (2)$$

The drying rate (DR) of the tea leaves was determined by the amount of moisture removed per unit of time, calculated using Equation (3)

$$DR = \frac{M_{t+dt} - M_t}{dt} \qquad (3)$$

Where: M_{t+dt} and M_t are the moisture contents at times t+dt and t, respectively; dt is the time interval between two measurements (min).

B. Mathematical Modelling and Numerical Simulation

The moisture diffusion process in tea leaves was described using a numerical simulation based on Fick's second law for a flat-plate geometry [7]. The partial differential equation (PDE) describes the variation of moisture concentration (X) over space (z - representing the coordinate along the thickness of the tea leaf) and time (t):

$$\frac{\partial X}{\partial t} = \frac{\partial}{\partial z}\left(D_{eff}(X,T)\frac{\partial X}{\partial z}\right) \qquad (4)$$

Where D_{eff} is the effective moisture diffusion coefficient (m^2/s), which is temperature-dependent according to the Arrhenius equation; this system of equations was solved using the Finite Difference Method (FDM) with an implicit scheme in MATLAB to ensure convergence and accurately describe the moisture concentration gradient from the core to the surface of the material.

$$D_{eff} = D_o.\exp\left(-\frac{E_a}{R.T}\right) \qquad (5)$$

Where: D_o is the pre-exponential factor m^2, E_a is the activation energy (J/mol), R is the universal gas constant (8314 J/mol.K), and T is the absolute temperature (K).

C. Thin-Layer Drying Models

To describe the drying kinetics of green tea leaves using a hybrid heat pump system integrated with auxiliary electric heating, six common mathematical models were applied to fit the experimental data [8] (Table I).

Table I: Thin-Layer Drying Models Applied in the Study

No.	Model Name	Model Equation	References
1	Newton	$MR = \exp(-kt)$	[8]
2	Wang and Singh	$MR = 1 + at + bt^2$	[10]
3	Page	$MR = \exp(-kt^n)$	[8]
4	Midilli and Kucuk	$MR = a.\exp(-k_1t^n) - \exp(-k_2t^n) - bt^n$	[9]
5	Two - term exponential	$MR = a.\exp(-kt) + (1-a).\exp(-kat)$	[10]
6	Verma	$MR = a.\exp(-kt) + (1-a).\exp(-gt)$	[10]

In these equations: t is the drying time (min); k and g are the drying rate constants (min^{-1}); a, b, and n are dimensionless model parameters related to the shape and diffusion mechanism of the tea leaves.

D. Statistical Evaluation of the Models

The goodness-of-fit between the mathematical models and the experimental data was evaluated using non-linear regression analysis. The primary statistical criteria included the coefficient of determination (R^2), the root mean square error (RMSE), and the reduced chi-square (χ^2). These values were calculated using Equations (6), (7), and (8):

Coefficient of determination (R^2)

$$R^2 = 1 - \frac{\sum_{i=1}^{N} (\overline{MR_{exp,i}} - MR_{pre,i})^2}{\sum_{i=1}^{n} (\overline{MR_{exp,i}} - MR_{avg})^2} \quad (6)$$

Root means square error (RMSE):

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N} (MR_{pre,i} - MR_{exp,i})^2}{N}} \quad (7)$$

Reduced chi-square (χ^2)

$$\chi^2 = \frac{\sum_{i=1}^{N} (MR_{exp,i} - MR_{pre,i})^2}{N - n} \quad (8)$$

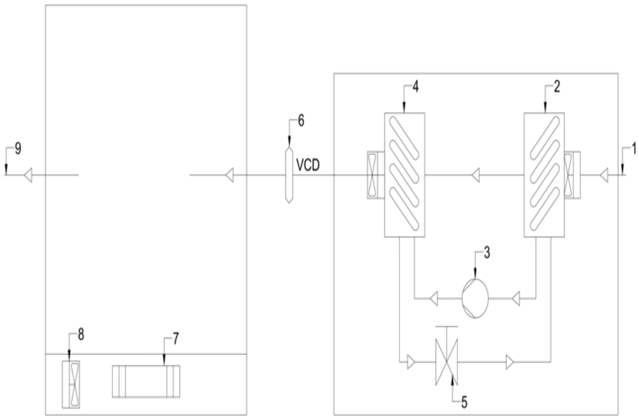
Where $MR_{exp,\,i}$ is the i_{th} experimental moisture ratio; $MR_{pre,\,i}$ is the i_{th} predicted moisture ratio from the model; $\overline{MR_{exp}}$ is the average experimental moisture ratio; N is the number of observations; and z is the number of constants in the model. A model is considered optimal when it yields the highest R^2 value (approaching 1) while simultaneously achieving the lowest RMSE and χ^2 values.

E. Experimental Setup

i. Drying Equipment and System Schematic

The drying system employed in this study is a heat pump dryer integrated with auxiliary electric heating, designed for flexible operation within a temperature range of 30 °C to 100 °C. The main components of the system include:

- **Heat Pump Unit:** Equipped with a 1.6 kW compressor using R134A refrigerant. This unit is responsible for dehumidification and providing a stable heat source during the initial drying stage.
- **Auxiliary Heater:** A 1.4 kW electric resistance heater installed at the bottom of the drying chamber to facilitate rapid increases in the drying agent temperature during specific stages as required.
- **Drying Chamber:** Featuring a capacity of 190L and a 0.136 kW centrifugal circulation fan to ensure uniformity of temperature and air velocity across the drying trays.



[Fig.2: Schematic Diagram of the Hybrid Heat Pump Drying System Integrated with Electric Heating]

1. Ambient air
2. Evaporator
3. Compressor
4. Condenser
5. Expansion valve
6. Flow control valve
7. Heater
8. Fan
9. Exhaust air

[Fig.3: 3D View and Actual Physical System of the Hybrid Heat Pump Drying System Integrated with Electric Heating]

Detailed technical specifications of the experimental system are presented in [Table II.](#)

Table II. Detailed Technical Specifications of the Experimental Drying System

No.	Parameter	Unit	Value
1	Capacity	Liters	190
2	Temperature adjustment range	ºC	30 - 100
3	Temperature resolution	ºC	1
4	Temperature control accuracy	%	± 1
5	External dimensions (L x W x H)	mm	760x520 x 800
6	Internal dimensions (L x W x H)	mm	700 x 450 x 600
7	Weight	kg	100
8	Rated dehumidification capacity (System)	kg	150kg/24h
9	Heat pump power	kW	1.6
10	Electric heater power	kW	1.4
11	Circulation fan power	kW	0.136
12	Total electrical power consumption	kW	3.136
13	Refrigerant		R134A
14	Power supply	220V, 50Hz ± 10%	

ii. Experimental Drying Modes

To evaluate the influence of combined heat sources on the moisture removal kinetics and the material's drying rate, experiments were conducted under three different control modes. A common feature across all three modes was the initial thermal stabilisation phase, lasting 150 minutes (2.5 hours), during which the system operated solely on the heat pump unit at a constant temperature of 35 °C. Maintaining this low temperature in the early stage was intended to protect the capillary structure of the fresh tea leaves, establishing a foundation for stable internal moisture diffusion in subsequent stages.

The experimental operating modes designed to investigate the effects of multi-stage temperature strategies are detailed in [Table III:](#)

Table III: Experimental Operating Modes and Drying Conditions

Mode	Dryer Operating Time		
	Stage 1 (Initial Stabilization)	Stage 2	Stage 3
Mode 1	Heat Pump: 2.5 hours at 35 °C	Hybrid (Heat Pump + Electric): 2 hours at 40 °C	
Mode 2	Heat Pump: 2.5 hours at 35 °C	Hybrid (Heat Pump + Electric): 1.5 hours at 45 °C	
Mode 3	Heat Pump: 2.5 hours at 35 °C	Hybrid (Heat Pump + Electric): 1 hour at 40 °C	Hybrid (Heat Pump + Electric): 2 hours at 45 °C

iii. Experimental Procedure

The experimental process was conducted sequentially through the following steps to ensure the accuracy of the drying kinetics data:

- **Step 1:** Sample Preparation: Fresh green tea leaves, after enzyme inactivation to stabilise their structure, were spread uniformly on drying trays at a loading density of approximately 0.5 kg/m² (equivalent to 5 g per quantitative experimental sample). This thin-layer distribution helps minimize errors caused by uneven heat and moisture distribution within the material layer.
- **Step 2:** Operation and Stabilization: The drying system was started 30 minutes in advance to bring the drying chamber to a stable thermodynamic state (35 °C), in accordance with the Stage 1 settings, before the samples were placed on the trays.
- **Step 3: Data Collection:**

- Mass: Recorded periodically every 30 minutes using a RADWAG PS 750.R1 precision balance with an accuracy of 0.001 g. This data served as the basis for calculating the experimental moisture ratio (MR_{exp})

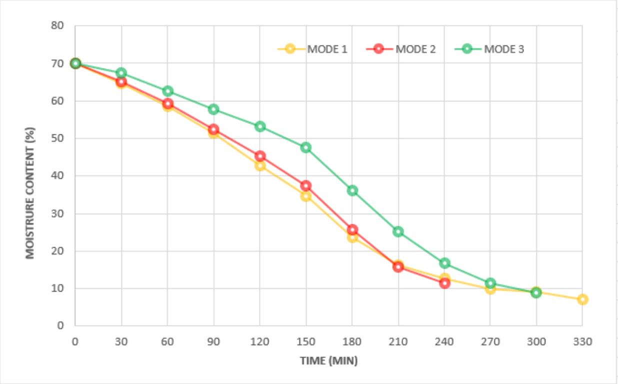
- Environmental Parameters: The temperature and humidity of the drying agent were continuously monitored via a high-precision sensor system connected directly to the HMI interface. These real-time temperature values were extracted as input parameters for the numerical simulation model in MATLAB.

- **Step 4:** Termination and Preservation: The drying process was terminated when the sample mass showed no significant change over three consecutive measurements or when the material reached the target moisture content range of 7% to 11% (wet basis), corresponding to approximately 0.075 to 0.124 kg/kg (dry basis). The conversion and control on a dry-basis (kg/kg db) ensured synchronisation between the experimental data and the variables in the Fickian diffusion and thin-layer kinetic models established in Section 2.2. Post-drying products were vacuum-sealed and stored under standard conditions for subsequent analysis and evaluation.

III. RESULT AND DISCUSSION

A. Analysis of Drying Curves and Drying Rate

The variations in the moisture content (%) of green tea leaves over the drying time (min) for the three operational modes are illustrated in Fig.4. The results demonstrate a clear influence of the temperature control strategy on the moisture removal kinetics:



[Fig.4: Variation of Moisture Ratio (MR) Over Drying Time for the Three Experimental Modes]

i. *Initial Stabilization Phase (0 – 150 min):* During the first 150 minutes, all three modes exhibited a relatively consistent moisture reduction trend. The moisture content decreased from approximately 70% to 35%–48%. Notably, Mode 3 (represented by the green curve) exhibited a slightly more gradual moisture reduction than Mode 1 and Mode 2, suggesting a more conservative heat application in the early stages to preserve the leaf structure.

ii. *Acceleration Phase via Auxiliary Heating (After 150 min):* A significant divergence in the drying curves was observed after the 150-minute mark, corresponding to the activation of the auxiliary electric heater:

iii. *Mode 2 (Red Curve):* Upon raising the temperature to 45°C at 150 minutes, the moisture content exhibited the sharpest decline. The curve displayed the highest steepness, achieving the target moisture content (below 11%) within 240 minutes. This mode achieves the highest drying-time efficiency, resulting in a 27.2% reduction in processing time compared to Mode 1.

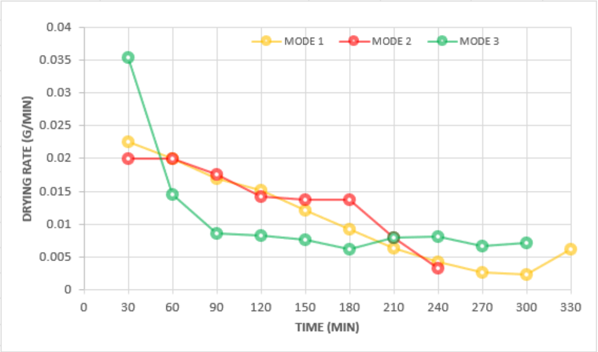
iv. *Mode 1 (Yellow Curve):* Although the drying rate was initially comparable, it significantly decelerated after 150 minutes (at 40°C), resulting in the longest total drying time of 330 minutes.

v. *Mode 3 (Green Curve):* Utilizing a multi-stage temperature strategy (40°C from 150–210 min and 45°C thereafter), the moisture reduction curve for Mode 3 was the most stable and smooth. Despite a slower onset, the activation at 210 minutes facilitated a robust acceleration in moisture removal, concluding the drying process at 300 minutes.

vi. *Scientific Discussion:* The experimental results confirm that raising the temperature to 45°C using the auxiliary electric heater increases the material's internal vapour pressure, thereby enhancing the moisture diffusion driving force. While Mode 2 provided the shortest drying time, Mode 3 (stepped temperature) demonstrated a more controlled drying process. This staged approach allows the leaf structure to gradually adapt to thermal changes, potentially minimizing structural shrinkage and preventing the "case hardening" phenomenon often associated with rapid moisture removal. Consequently, Mode 3 offers an optimal balance between process efficiency and the preservation of product quality.

B. Drying Rate Analysis

The experimental drying rate (DR) curves for the three modes provide critical insights into the moisture removal efficiency of the hybrid heat pump system:



[Fig.5: Drying Rate Curves of Green Tea Leaves Under Different Experimental Modes]

- Initial Phase (0 – 150 min):

  i. *Mode 1 & 2:* Display a steady and gradual decline in drying rate. This is typical of the falling-rate period, where the drying process is limited by internal moisture diffusion.

  ii. *Mode 3 (Green Curve):* Shows an exceptionally high initial drying rate (>0.035 g/min) that then drops sharply. This "peak" at the 30-minute mark suggests a rapid removal of surface free moisture. After 90 minutes, Mode 3 maintains a lower but more stable drying rate than the other modes during the 35 °C stabilisation phase.

- Impact of Auxiliary Heating (After 150 min)

  iii. *Mode 2 (Red Curve):* This is the most significant observation. Between 150 and 210 minutes, while other modes continue to decline, Mode 2 exhibits a "plateau" or a slight secondary peak in drying rate. This is the direct result of activating the auxiliary heater at 45 ºC. The additional thermal energy compensates for the increased internal resistance, maintaining a high DR (0.014 g/min) and leading to the fastest completion of the drying process.

  iv. *Mode 3 (Green curve):* Shows a distinct increase in DR at the 210-minute mark. This corresponds to the second temperature step (from 40$^{\circ\mathrm{C\ to\ }45^{\circ}\mathrm{C}}$). Unlike Mode 2, which had a single high burst, Mode 3 shows a "rebound" effect, maintaining a higher DR in the final stages (210-300 min) than in Mode 1 or Mode 2.

  v. *Final Phase:* As the moisture content reaches the target range, all modes converge towards a minimum DR. In Mode 1, the DR continues to tail off slowly, confirming that without sufficient auxiliary heat, the final removal of bound water becomes extremely slow and energy-intensive.

C. Results of Mathematical Modelling

After performing non-linear regression for the six common thin-layer drying models, the statistical parameters—including the coefficient of determination R^{2,} root mean square error (RMSE), and reduced chi-square χ^2 —were synthesized in **Table-IV.** to evaluate the goodness-of-fit.

Table IV: Statistical Analysis Results for the Thin-Layer Drying Models

Mode	Model No.	Model Constants	r	R^2	χ^2	RMSE
1	1	k = 0.00550	0.98230	0.93670	0.00708	0.08053
	2	a = -0.00400, b = 3.41x10^{-6}	0.99050	0.97740	0.00280	0.04810
	3	k = 0.00020, n=1.62240	0.99760	0.99490	0.00062	0.02278
	4	a = 2.00000, k_1 = 0.00220, n = 1.26360, k_2= 0.00370, b = 2.33x10^{-14}	0.99780	0.99550	0.00078	0.02140
	5	a = 2.07530, k = 0.00930	0.99630	0.99250	0.00093	0.02780
	6	a = 0.50360, k = 0.00550, g = 0.00560	0.98230	0.93670	0.00860	0.08050
2	1	k = 0.00490	0.96510	0.89440	0.00980	0.09350
	2	a = -0.00240, b = -5.17x10^{-6}	0.99670	0.99330	0.00071	0.02350
	3	k = 0.00005, n =1.90010	0.99610	0.99140	0.00091	0.02660
	4	a = 2.00000, k_1 = 0.00048, n = 1.46430, k_2 = 0.00076 b = 0.00094	0.99810	0.99630	0.00072	0.01790
	5	a = 2.10520, k = 0.00910	0.99840	0.97600	0.00250	0.04450
	6	a = 0.44690, k = 0.00490, g = 0.00500	0.96510	0.8944	0.01310	0.09350
3	1	k = 0.00420	0.95390	0.85360	0.01570	0.11930
	2	a = -0.00170, b = 4.72x10^{-6}	0.99080	0.98110	0.00220	0.04290
	3	k = 0.0001, n =2.23270	0.99650	0.99220	0.00093	0.02760
	4	a = 2.00000, k_1 = 0.00060, n =1.69020, k_2 = 0.00002 b = 8.37x10^{-7}	0.99510	0.99440	0.00062	0.01840
	5	a = 1.88570, k = 0.00590	0.97310	0.94600	0.00390	0.05860
	6	a = 0.30780, k = 0.00420 g = 0.00420	0.95390	0.85360	0.01960	0.11930

D. Comparative Performance of the Models

- Best-fit Models: Across all three experimental modes, the Midilli and Kucuk model (Model 4) and the Page model (Model 3) consistently outperformed the others.

In Mode 2 and Mode 3, Model 4 yielded the highest R^2 values (0.9981 and 0.9951, respectively) and the lowest RMSE (0.0179 and 0.0184).

In Mode 1, both Model 3 and Model 4 showed excellent alignment with $R^2 > 0.997$.

- Poor-fit Models: The Newton model (Model 1) and the Verma model (Model 6) exhibited the lowest compatibility, particularly in Mode 2 and Mode 3 (R^2 ranging from

Retrieval Number: 100.1/ijpte.D203006040626
DOI: 10.54105/ijpte.D2030.06040626
Journal Website: www.ijpte.latticescipub.com

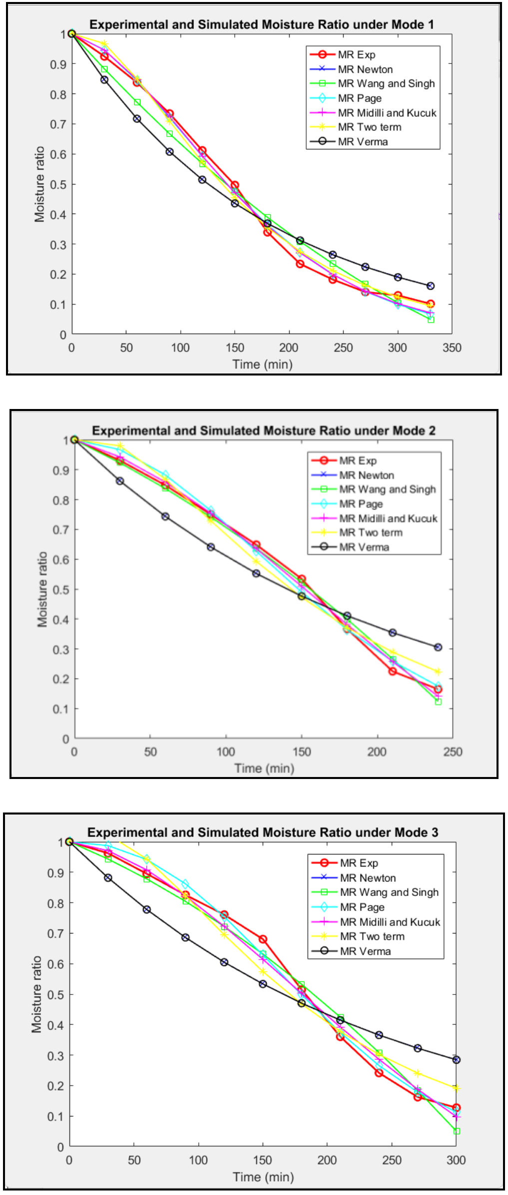
Published By:
Lattice Science Publication (LSP)

0.9539 to 0.9651). This indicates that a simple exponential model is inadequate for describing the complex moisture migration in green tea leaves under hybrid heating and variable-temperature stages.

E. Influence of Drying Modes on Model Constants

Drying Constant (*k*): The values of *k* in the Page and Midilli-Kucuk models reflect the drying rate. In Mode 1, k was relatively higher, whereas in Modes 2 and 3, the interaction between *k* and the empirical exponent *n* describes a more complex diffusion path.

Empirical Exponent (*n*): The parameter *n* in the Page model increased from 1.6224 (Mode 1) to 2.2327 (Mode 3). This upward trend suggests that the stepped-temperature strategy significantly modifies the internal diffusion mechanism, likely by reducing internal resistance and enhancing capillary moisture flow within the tea leaf matrix.



[Fig.6: Correlation Between Experimental Moisture Ratio (MR) and Predicted Values from the Midilli and Kucuk Model]

Fig.6 compares the experimental moisture ratio (MRexp) with the predicted values (MR_{pre}) obtained from the Midilli and Kucuk model. The data points are closely distributed around the 45 ° line (Y = X), indicating excellent agreement between the observed and simulated results. The high concentration of data points along the diagonal line further confirms that the Midilli and Kucuk model accurately describes the drying behaviour of green tea leaves across various hybrid heat pump operating modes.

F. Determination of Effective Moisture Diffusivity (Deff)

The effective moisture diffusivity, Deff, is a key parameter that reflects the mobility of water molecules from the interior of the tea leaves to the surface. Based on the Partial Differential Equation (PDE) model and the Arrhenius-type relationship, the D_{eff} values at the investigated temperatures are summarised in Table V.

Table V: Effective Moisture Diffusivity of Green Tea Leaves at Different Drying Temperatures

Drying Temperature (°C)	Heating Mode	D_{eff} (m2/s)	R^2
35	Heat Pump (HP)	1.24E10^{-11}	0.9854
40	Hybrid (HP + Electric)	1.86E10^{-11}	0.9912
45	Hybrid (HP + Electric)	2.51E10^{-11}	0.9945

The results in Table 5 indicate that D_{eff} values increased significantly with rising drying temperatures, ranging from 1.24E10^{-11} to 2.51E10^{-11} m^2/s. This trend is consistent with the kinetic theory of gases and liquids, in which higher thermal energy increases the vibrational motion of water molecules, thereby reducing the internal resistance to mass transfer.

Notably, the D_{eff} at 45 °C was approximately 2.02 times higher than at 35 °C. This substantial increase confirms the effectiveness of the auxiliary electric heater in accelerating the internal moisture migration. The calculated D_{eff} values for Thai Nguyen green tea in this study fall within the typical range for agricultural products (10^{-12} to 10^{-8} m^2/s), verifying the reliability of the experimental and numerical methods employed.

Justification for the Midilli and Kucuk Model

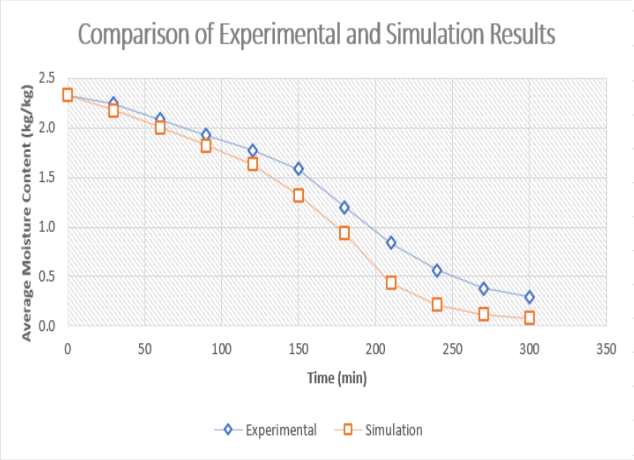
The superior performance of the Midilli and Kucuk model in the hybrid heat pump system can be attributed to its four-parameter structure (*a, k, n, b*). The linear term (*bt*) in this model effectively accounts for the constant-rate-like behaviour sometimes observed during the initial transition between heat pump and electric-heating stages. At the same time, the exponential component captures the primary falling-rate diffusion.

G. Validation with Numerical Simulation (MATLAB)

To evaluate the reliability and practical applicability of the established mathematical model, the numerical solutions obtained from MATLAB were directly validated against the experimental data. Among the investigated strategies, Mode 3 (multi-stage temperature profile: 35 °C for 150 min, 40 °C for 60 min, and 45 °C until completion) was selected for validation. This mode represents the most complex drying kinetics and has been identified as the optimal strategy for balancing drying efficiency and product quality.

i. Comparison of Experimental and Simulation Curves

The comparison between the experimental moisture content (X_{exp}) and the simulated values (X_{sim}) for Mode 3 is illustrated in Fig.7.

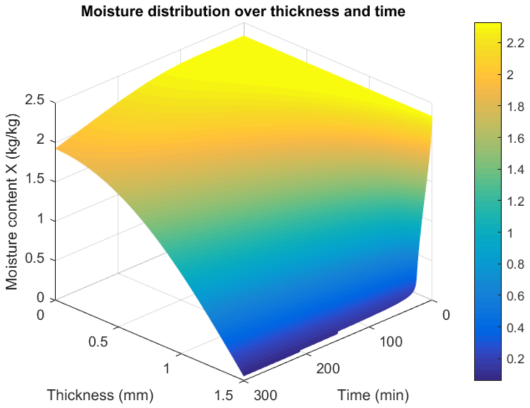


[Fig.7: Comparison of Experimental Average Moisture Content and Numerical Simulation Results (X_{pre}) for the Multi-Stage Drying Mode (Mode 3)]

- Model Accuracy: The numerical results show a high degree of agreement with the experimental data, successfully capturing the "inflexion points" at 150 and 210 minutes when the temperature was shifted. The Finite Difference Method (FDM) effectively simulates the accelerated moisture removal triggered by the auxiliary electric heater.
- Error Analysis: The slight deviation observed during the transition phases (RMSE < 0.05) can be attributed to the thermal inertia of the drying system and the non-homogeneous nature of the tea leaf bed. However, the overall alignment confirms that the 1D diffusion model is a robust tool for predicting drying behaviour under transient thermal conditions.

ii. Spatiotemporal Moisture Distribution

The 3D moisture distribution profile (spatial and temporal) generated by the MATLAB simulation provides a deeper understanding of the internal mass transfer mechanisms (Fig. 8).



[Fig.8: Simulated 3D Spatiotemporal Moisture Distribution Within the tea leaf Matrix During the Mode 3 Drying Process]

- Internal Gradients: The model reveals a clear moisture gradient between the core ($z = 0$) and the surface ($z = L$) of the tea leaves. This gradient acts as the primary driving force for diffusion.
- Effect of Staged Heating: The 3D surface shows a distinct change in slope following each temperature increase. By simulating these gradients, the model demonstrates how Mode 3 maintains a consistent drying driving force without subjecting the material to excessive surface over-drying, which is critical for preserving heat-sensitive compounds like EGCG in green tea.
- Conclusion of Validation: The successful validation of the most complex operational mode (Mode 3) confirms that the developed simulation tool is highly reliable. It can accurately predict drying time and internal moisture levels, significantly reducing the need for trial-and-error in optimising hybrid heat pump drying processes.

IV. CONCLUSION

This study successfully investigated the drying kinetics of green tea leaves using a hybrid heat pump system integrated with auxiliary electric heating. The key findings and contributions are summarized as follows:

A. Performance of Hybrid Drying Strategies: The experimental results demonstrate that the application of auxiliary heating significantly enhances the drying process. Among the tested modes, Mode 3 (a multi-stage temperature strategy: 35 °C – 40 °C – 45 °C) was identified as the optimal approach. It balances a high drying rate with a controlled moisture gradient, effectively preventing "case hardening" and preserving the structural quality of the tea leaves compared to constant high-temperature strategies.

B. Mathematical Modelling and Kinetics: Among the six thin-layer drying models evaluated, the Midilli and Kucuk model exhibited the highest statistical accuracy in describing the moisture loss kinetics, with R^2 values consistently above 0.995 and minimal RMSE. The effective moisture diffusivity (D_{eff}) was found to increase with temperature, ranging from 1.24E-10 to 2.51E-10 m^2/s, which is characteristic of internal diffusion-controlled drying processes in agricultural materials.

C. Numerical Simulation and Applicability: The numerical simulation performed in MATLAB, utilizing the Finite Difference Method (FDM) to solve the 1D moisture diffusion equation, showed excellent agreement with experimental data (RMSE < 0.05). The model not only accurately predicts moisture loss over time but also provides detailed insights into the spatiotemporal moisture distribution within the tea leaf matrix.

D. Practical Implications: The developed simulation tool serves as a reliable decision-support system for process engineers. It allows for the optimization of operating parameters (temperature scheduling and drying duration) to maximize throughput and energy efficiency without the necessity of extensive experimental testing. Future research could integrate energy consumption analysis and quality assessment (e.g., EGCG retention) to further optimise the hybrid drying system for industrial-scale tea production.

APPENDIX

Thanks to the editors and reviewers for their support and comments on my article.

ACKNOWLEDGMENT

This study is the result of a student research project, funded by Hanoi University of Industry. The authors also wish to express their gratitude to the Faculty of Energy Engineering at Hanoi University of Industry and the Institute of Energy Science and Environment (IESE) for their technical support and for providing the experimental facilities required for this research.

DECLARATION STATEMENT

Some of the references cited are outdated, noted explicitly as [9] and [10]. However, these works remain significant for the current study, as they are pioneering in their fields.

Authors of review-type articles are required to include a declaration of accountability in the article that stipulates each author's involvement. The level of detail differs; Some subjects yield articles that consist of isolated efforts that are easily voiced in detail, while other areas function as group efforts at all stages. It should be after the conclusion and before the references.

As the article's author, I must verify the accuracy of the following information after aggregating input from all authors.

- **Conflicts of Interest/ Competing Interests:** Based on my understanding, this article has no conflicts of interest.
- **Funding Support:** This article has not been funded by any organizations or agencies. This independence ensures that the research is conducted objectively and without external influence.
- **Ethical Approval and Consent to Participate:** The content of this article does not necessitate ethical approval or consent to participate with supporting documentation.
- **Data Access Statement and Material Availability:** The adequate resources of this article are publicly accessible.
- **Author's Contributions:** Authors should describe the role of each author if there are more than 01 author. All authors have individual contributions in this article. The specific contributions are as follows: - Ngo Quang Truong: Conceptualisation, Methodology, Experimental setup, Data collection, and Drafting of the original manuscript, Software implementation (MATLAB), Numerical simulation, - Nguyen Thi Viet Linh: Project administration, Supervision, Validation of numerical models, Formal analysis, and Critical revision of the manuscript for important intellectual content. - Pham The Vu: Experimental setup, Data collection, and Technical support for the hybrid heating system configuration. - Pham Van Duy: Visualisation, Experimental setup, Data collection, and Technical support for the hybrid heating system configuration.and Technical support for the hybrid heating system configuration.

REFERENCES

1. X. Zhao, P. Yu, N. Zhong, H. Huang, and H. Zheng, "Impact of Storage Temperature on Green Tea Quality: Insights from Sensory Analysis and Chemical Composition," *Beverages*, vol. 10, no. 2, p. 35, 2024. DOI: https://doi.org/10.3390/beverages10020035
2. Y. Wang, Y. Liu, J. Huo, X. Zhao, J. Zheng, and X. Wei, "Effect of different drying methods on chemical composition and bioactivity of tea polysaccharides," *International Journal of Biological Macromolecules*, vol. 63, pp. 216–220, 2014. DOI: https://doi.org/10.1016/j.ijbiomac.2013.10.006
3. A. B. T. Loemba, B. Kichonge, and T. Kivevele, "Comprehensive assessment of heat pump dryers for drying agricultural products," *Energy Science & Engineering*, vol. 11, pp. 2985–3014, 2023. DOI: https://doi.org/10.1002/ese3.1326
4. Zhixia Sun et al., Recent Advances in Heat Pump Drying System for the Food Industry: Structure, System Optimisation, Applications, and Prospects*,* 26 July 2025, DOI: https://doi.org/10.1111/1541-4337.70238
5. N. V. May, *Kỹ thuật sấy nông sản thực phẩm*. Hanoi, Vietnam: Science and Technics Publishing House, 2004. https://fr.scribd.com/document/685865673/Giao-Trinh-K%E1%BB%B9-Thu%E1%BA%ADt-S%E1%BA%A5y-Nong-S%E1%BA%A3n-Th%E1%BB%B1c-Ph%E1%BA%A9m-Nguy%E1%BB%85n-V%C4%83n-May-658387-1
6. V. P. Tran, *Tính toán và thiết kế hệ thống sấy*. Hanoi, Vietnam: Education Publishing House, 2008. https://fr.scribd.com/document/366477746/Ebook-Tinh-Toan-Va-Thi%E1%BA%BFt-K%E1%BA%BF-H%E1%BB%87-Th%E1%BB%91ng-S%E1%BA%A5y-PGS-tskh-Tr%E1%BA%A7n-V%C4%83n-Phu
7. D. I. Onwude, N. Hashim, R. B. Janius, N. M. Nawi, and K. Abdan, "Modelling the Thin-Layer Drying of Fruits and Vegetables: A Review," *Comprehensive Reviews in Food Science and Food Safety*, vol. 15, no. 3, pp. 599-618, 2016. DOI: https://doi.org/10.1111/1541-4337.12196
8. A. Midilli and H. Kucuk, "Development of a new curve equation representing thin layer drying process," *Energy Sources, Part A: Recovery, Utilization, and Environmental Effects*, vol. 45, no. 4, pp. 9717–9730, 2023. DOI: https://doi.org/10.1080/15567036.2023.2240740
9. Oktay Hacıhafızoglu, Ahmet Cihan Kamil Kahveci, *Mathematical modelling of drying of thin-layer rough rice*, Food and bioproducts processing 86 (2008) 268–275, 2008. DOI: https://doi.org/10.1016/J.FBP.2008.01.002
10. Partha Pratim Dutta, Debendra Chandra Baruah, Drying modelling and experimentation of Assam black tea (Camellia sinensis) with producer gas as a fuel, Volume 63, Issue 2, 22 February 2014, Pages 495-502, 2014. DOI: https://doi.org/10.1016/j.applthermaleng.2013.11.035

AUTHOR'S PROFILE

Ngo Quang Truong was born on October 23, 2005, in Hai Phong, Vietnam. He graduated from Thuy Huong High School, and he is currently a third-year student at Hanoi University of Industry. Moreover, he studies in the Faculty of Energy, School of Electrical and Electronics Engineering, majoring in Thermal Engineering Technology, and he has been enrolled there since autumn 2023. In addition, he has actively participated in various student scientific research competitions; for example, his projects focus on heat pump drying systems, mini robots, and Robocon. Furthermore, he is eager to improve his technical knowledge and teamwork skills. Therefore, he aims to gain more practical experience and prepare himself for a future career in engineering.

Nguyen Thi Viet Linh, born on November 11, 1988, in Hai Duong, Vietnam, graduated with an engineering degree in 2011 from Hanoi University of Industry and a Master's degree in 2014 from Hanoi University of Science and Technology. I work at the Department of Energy, School of Electrical and Electronics Engineering, Hanoi University of Industry. I have been teaching and researching at the school since 2019. My major has been in thermal engineering technology. I have 15 years of experience in thermal technology and equipment. I have researched and collaborated with research groups in areas such as evaluating the replacement of refrigerants in refrigeration systems from banned refrigerants to environmentally friendly refrigerants; heat pump drying systems with integrated solar energy; biomass furnaces; and Fluidised bed drying systems. I have written reference books such as Refrigeration Safety and the Practice of Civil Air Conditioning.

Pham The Vu, born on April 09, 1977, in Vietnam, graduated with an engineering degree in 2006 from Nha Trang University, a Master's degree in 2010 from Hanoi University of Science and Technology and a PhD in 2022 at the Academy of Social Sciences. I work at the Department of Energy, School of Electrical and Electronics Engineering, Hanoi University of Industry. Since 2000, He has been a lecturer and researcher at the Department of Energy, School of Electrical and Electronics Engineering, Hanoi University of Industry. His primary field of expertise is thermal engineering technology. His research interests include evaluating alternative refrigerants to replace environmentally harmful substances in refrigeration systems; developing solar-powered adsorption refrigeration systems; and designing and optimising heat pump drying systems integrated with solar energy. He has actively collaborated with various research groups and contributed to advancing sustainable energy solutions. In addition to his research activities, he is the author of several reference textbooks used in teaching and academic research, including thermal safety, refrigeration system operation, and residential air conditioning practice. He was an expert in the world refrigeration competition.

Pham Van Duy was born in Nam Dinh, Vietnam, in 1982. He received his B.S. degree in Electronics and Telecommunications Engineering in 2003 and his M.S. degree in Electronic Engineering in 2011, both from the Hanoi Open University. With over 18 years of professional experience in the energy sector, he has profound expertise in energy auditing, energy efficiency projects, and renewable energy technologies. From 2007 to 2022, he served at the Centre for New and Renewable Energy at the Institute of Energy Science. Since 2022, he has been working at the Department of Sustainable Energy at the Institute of Energy Science and Environment (IESE) of the Vietnam Academy of Science and Technology. His current research interests focus on advanced energy-saving drying technologies, specifically on hybrid heat pump drying systems and freeze-drying (lyophilisation). He has successfully participated in and managed numerous high-level projects focused on the practical implementation of renewable and sustainable energy solutions.